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Vibro-Acoustics of a Brake Rotor with Focus on Squeal Noise

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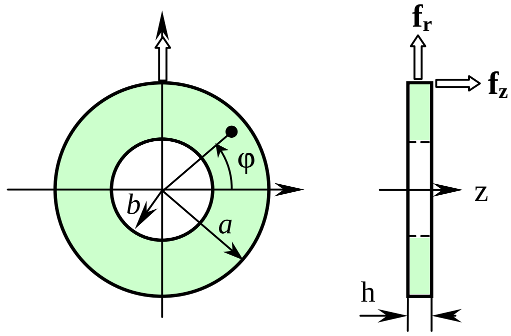
Abstract

Brake squeal has been a major noise problem in many ground vehicles and yet many issues remain poorly understood. One unresolved problem is the sound radiation mechanism. This article investigates this problem via a generic rotor example and determines the contributions of out-of-plane (flexural) and in-plane (radial) modes of the rotor to total sound radiation given harmonic excitations from 1 to 8 kHz. A new semi-analytical procedure is applied to solve this problem. In the proposed procedure, the disk surface velocities at selected modes are first defined using a finite element model and then acoustic radiation modes are obtained from the velocities via new analytical solutions that employ modified circular and cylindrical radiator models. Sound radiation due to multi-modal excitations is calculated using the modal expansion technique. In addition, acoustic power and radiation efficiency spectra corresponding to a specific force excitation are obtained from far-field sound pressure data. Analytical predictions are confirmed by comparisons with numerical and/or experimental investigations

1. Introduction

The vehicle brake squeal problem has been investigated by using the complex mode method as the unstable elastic modes of deformation are likely to generate squeal noise from 1 to 16 kHz [1-2]. One of the hypotheses is that the dynamic coupling between in-plane and out-of-plane modes of brake rotor leads to an instability that produces intense sound at several high frequencies. Most of the prior studies have focused on the structural dynamics of brake rotors and related components and to the best of our knowledge, no one has adequately examined the acoustic radiation mechanism. In this article, sound radiation from a brake rotor, the main squeal radiator, is investigated using a thick annular disk example. A new semi-analytical approach, based on numerically obtained disk surface velocities and analytical modal sound radiation solutions, will be introduced. The validity of this procedure has been confirmed by computational analyses and vibro-acoustic experiments. Figure 1 describes the geometry and material properties of the example disk. For a complete investigation of the vibro-acoustic characteristics of a thick annular disk, we simultaneously consider in-plane and out-of-plane vibrations.



Dimensions and Material Properties	
Outer radius (a)	151.5 mm
Inner radius (b)	87.5 mm
Thickness (h)	31.5 mm
Density (ρ_d)	7905.9 Kg/m^3
Young's Modulus (E)	218 GPa
Poisson's Ratio (ν)	0.305

Figure 1: Dimensions and material properties of a thick annular disk.

The scope of this study is strictly limited to the frequency domain analysis of a linear time-invariant (LTI) system with free-free boundaries. Complicating effects such as fluid loading and scattering at the disk edges are not considered. Primary assumptions are as follows: (1) For out-of-plane modes, structural velocities in the normal direction (z) vary sinusoidally in the φ direction. (2) For the radial modes, velocities on the two radial surfaces are uniform in the z direction but vary sinusoidally in the φ direction. Chief objectives of this article are: (1) Introduce new sound radiation solutions for the normal modes of a thick annular disk. (2) Develop a semi-analytical procedure for calculating sound radiation from the disk given multi-modal harmonic excitations. The new method should allow us to examine the couplings between in-plane and out-of-plane modes as well as couplings within the same type of modes as they have considerable effects on the total sound radiation.

2. Structural Analysis for a Thick Annular Disk

As the first step, both out-of-plane and in-plane modal vibrations of a thick annular disk are investigated using a finite element model (FEM) that consists of 4,400 solid elements and 6,600 nodes [3]. Numerical results are checked with analytical solution based on the thick plate theory for out-of-plane modes [4-5] and the transfer matrix method for radial modes [6-7] along with experimental modal analyses. The results are summarized in Table 1 where λ_{mn} and λ_q are dimensionless eigenvalues for the (m, n)th out-of-plane and the qth in-plane modes. Finite element based predictions match quite well with analytical and experimental results and these can be used for forced vibration response and acoustic radiation calculations.

Table 1: Comparison of natural frequencies for a thick disk with free-free boundaries

Mode Type	Mode Indices	ω_{mn} or ω_q (Hz)		
		Measured	Computed (FEM)	Analytical
Out-of-Plane (m, n) $\omega_{mn} = \lambda_{mn}^2/a^2(\rho_d h/D_b)^{1/2}$	0 2	1331	1307	1328
	1 0	3063	2946	3077
	0 3	3481	3682	3682
In-Plane (q) $\omega_q = \lambda_q/(\rho_d ha^2/D)^{1/2}$	2	2831	2811	2854
	3	6869	6817	6915
	0	7266	7210	7509

3. Modal Acoustic Radiation Solutions

Sound radiation from the out-of-plane vibrations of circular and annular plates has been examined using the Rayleigh integral method, the impedance approach, and the 2-D Fourier transform technique [8-10]. Recently, we developed an analytical solution for sound radiation from the out-of-plane modes of a thick annular disk considering the thickness effect and expressed far-field sound pressure in terms of modal vibrations as follows [10] (See Figure 2). Here, m and n are the numbers of nodal circles and diameters, ρ is the mass density of air, c is the speed of sound, k_{mn} is the acoustic wave number, $\dot{W}_{mn}$ is the velocity amplitude at $\mathbf{r}_s$, and J_n is the Bessel function of order n.

$$P_{mn}(R,\theta,\phi) = \frac{\rho c k_{mn} e^{ik_{mn}R}}{R} e^{-ik_{mn}\frac{h}{2}\cos\theta} \cos n\phi (-i)^{n+1} \int_0^\infty \dot{W}_{mn}(r) J_n(k_{mn}\sin\theta r) r dr \qquad (1)$$

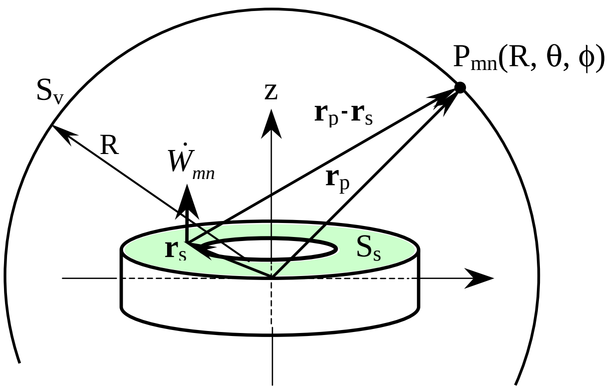


Figure 2: Radiation from an out-of-plane mode in the spherical coordinate system.

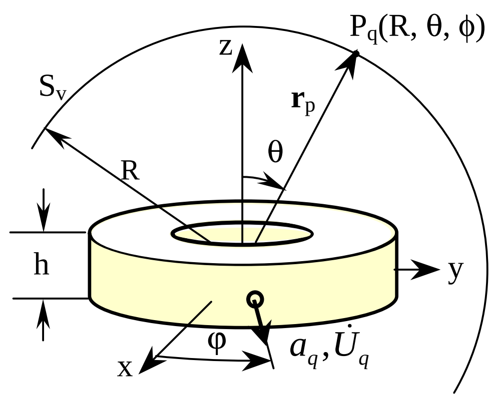


Figure 3: Radiation from an in-plane mode in the spherical coordinate system.

Also, we have developed expressions for far-field sound radiation from radial modes using a cylindrical radiator model [7]. Sound pressures are generated by two radial surfaces of the disk. The total sound pressure is then expressed as a summation of P_{qO} from the outer radial surface and P_{qI} from the inner radial surface (See Figure 3). Here, q is the radial mode index, k_q is the corresponding acoustic wave number, H_q is the Hankel function of order q, and $|a_q|$ is the acceleration amplitude of the on the radial edges due to the q^{th} radial mode. Also, the Sinc function is defined as Sinc(ξ) = sin(ξ) / ξ.

$$P_q(R,\theta,\phi) = P_{qI}(R,\theta,\phi) + P_{qO}(R,\theta,\phi)$$

$$P_{qO}(R,\theta,\phi) = \frac{\rho e^{ik_q R}}{\pi k_q R \sin\theta} |a_q| h \frac{Sinc(k_q \sin\theta\, h/2)(-i)^{q+1}}{H_q^{1\prime}(k_q a \sin\theta)} \cos q\phi \qquad (2a - c)$$

$$P_{qI}(R,\theta,\phi) = \frac{\rho e^{ik_q R}}{\pi k_q R \sin\theta} |a_q| h \frac{Sinc(k_q \sin\theta\, h/2)(-i)^{q+1}}{H_q^{2\prime}(k_q b \sin\theta)} \cos q\phi$$

Modal acoustic radiation solutions have been confirmed by numerical and experimental investigations with the sample disk in terms of modal acoustic power (Π), modal radiation efficiency (σ), and directivity patterns. The computational boundary element (BEM) model is constructed using 6,146 acoustic field points and 6,144 elements that are defined on the sphere surrounding the disk that is represented by the finite element model for structural dynamics [11]. The center of this sphere coincides with the disk center. Typical results are summarized in Table 2 and Figure 4. Analytical methods produce sufficiently accurate modal radiation solutions and thus these are used for further calculations.

Table 2: Comparison of modal acoustic power and radiation efficiency for selected modes

Mode Type		Π (dB re 1 pW)			σ		
		Measured	Computed (BEM)	Analytical	Measured	Computed (BEM)	Analytical
In-Plane	2	60.0	66.5	66.0	0.14	0.63	0.63
	3	62.0	67.5	67.5	0.22	0.89	0.79
Out-of-Plane	0, 3	70.6	76.3	76.2	0.14	1.01	1.01
	1, 1	67.3	70.6	71.6	0.20	0.75	1.08

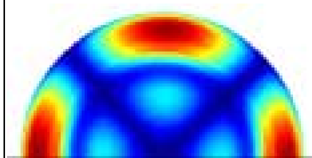
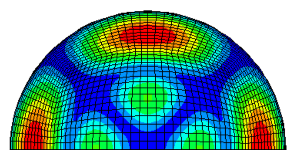
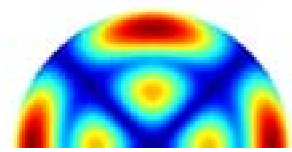
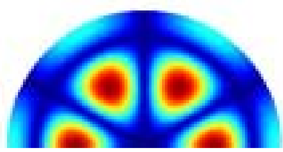
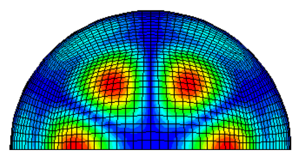
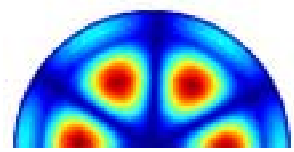
Mode	Measured	Computed (BEM)	Analytical
In-Plane q = 2			
Out-of-Plane			

Figure 4: Directivity patterns for selected structural modes.

4. Effect of Multi-Modal Excitations on Sound Radiation

If a disk is excited by a harmonic force vector consisting of one or more frequencies, several in-plane and out-of-plane modes are simultaneously excited. Based on the modal expansion technique, velocity distribution (**v**) on the normal and radial surfaces can be expressed in terms of the normalized elastic modes of this disk. Also, the far-field sound pressure is assumed to be determined by the modal sound radiation solutions of Section 3.

$$\begin{aligned} &\{v\}=\{\eta\}^T\{\Phi\} \quad \text{and} \quad P=\{\eta\}^T\{P\} \\ &\{\eta\}=\{\eta_{0,2,-1},\eta_{1,0,-1},\eta_{0,3,-1},\cdots,\eta_{m,n,-1},\eta_{-1,-1,2},\eta_{-1,-1,3},\cdots,\eta_{-1,-1,q}\} \\ &\{\Phi\}=\{\Phi_{0,2,-1},\Phi_{1,0,-1},\Phi_{0,3,-1},\cdots,\Phi_{m,n,-1},\Phi_{-1,-1,2},\Phi_{-1,-1,3},\cdots,\Phi_{-1,-1,q}\} \\ &\{P\}=\{P_{0,2,-1},P_{1,0,-1},P_{0,3,-1},\cdots,P_{m,n,-1},P_{-1,-1,2},P_{-1,-1,3},\cdots,P_{-1,-1,q}\} \end{aligned} \tag{3a - e}$$

where $\eta_{m,n,q}$ is the modal participation factor, $\Phi_{m,n,q}$ is a modal vector of the disk, and $P_{m,n,q}$ is the modal radiation solution for a specific mode. In this expression, modal indices (m, n, q) combine the out-of-plane mode indices (m, n) and radial mode index q. The –1 value in the indices is used to represents a null. For instance, $\Phi_{0,2,-1}$ is the out-of-plane mode with only two radial diameters and $\Phi_{-1,-1,2}$ is the q = 2 pure radial mode. Modal participation factors due to a harmonic excitation at frequency ω is calculated from the modal data set as follows:

$$\eta_{m,n,q}=\sum\frac{\Phi_{m,n,q}(r_f,\varphi_f)\Phi_{m,n,q}{}^T(r,\varphi)}{(1-\omega^2/\omega_{m,n,q}^2)+i2\varsigma_{m,n,q}(\omega/\omega_{m,n,q})} \tag{4}$$

Further, Π and σ due to an arbitrary force f(t) is calculated from the far-field acoustic intensity (I) or sound pressures on a sphere (S_v) surrounding the disk as follows:

$$\Pi = \langle IS_v \rangle_s = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} \frac{P^H P}{\rho_0 c_0} R^2 \sin\theta \, d\theta \, d\phi; \quad \sigma = \frac{\Pi}{<|\dot{v}|^2>_{t,s}} \qquad \text{(5a - b)}$$

Here $<|\dot{v}|^2>_{t,s}$ is the spatially averaged mean-square velocity on the radiating surfaces and it is expressed as follows.

$$<|\dot{v}|^2>_{t,s} = \frac{1}{4\pi h(a+b) + 2\pi(a^2 - b^2)} \left\{ \int_{-h/2}^{h/2}\int_0^{2\pi(a+b)} \dot{U}^2 dl\, dz + \int_b^a \int_0^{2\pi} \dot{W}^2 d\varphi\, dr \right\} \qquad (6)$$

Fundamental radiation properties such as sound pressure spectra P(ω), acoustic power spectra Π(ω), and radiation efficiency spectra σ(ω) of the sample annular disk are calculated given unit amplitude force. Also, the same radiation properties are obtained with numerical (FEM and BEM) analyses and vibro-acoustic experiments. Figure 5 shows P(ω) at two $\mathbf{r}_p$ locations (R = 303 mm, ϕ = 0, θ = 0; π/2) due to an excitation in the z direction. In both cases, excellent agreements between analytical and numerical results are found, especially in the vicinities of the peaks corresponding to natural frequencies of the disk. Figure 5b shows a significant coupling between in-plane and out-of-plane modes in measured spectrum. This is due to an imprecise force excitation in experiments. Further, note that P(ω) depend on the receiver positions since the source is highly directive. In addition, Π(ω) and σ(ω) for the combined (normal + radial) excitation are calculated using the proposed analytical procedure and compared with corresponding numerical results in Figure 6. Every peak in Π(ω) corresponds to a specific natural frequency. As evident from Figures 5 and 6, all analytical results match well with numerical (FEM + BEM) results over the given frequency range.

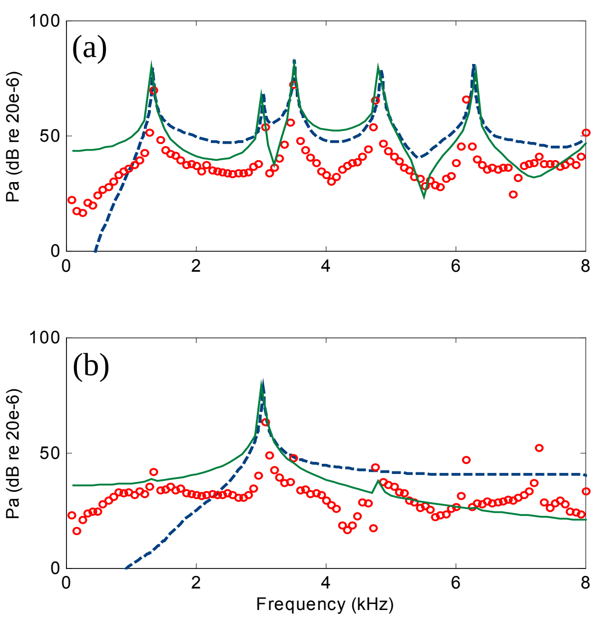


Figure 5: P(ω) due to force f_z = 1N. (a) θ = π/2 and ϕ = 0; (b) θ = 0 and ϕ = 0. Key: ∘∘∘∘, measured; —, analytical calculation; ----, computed (FEM + BEM).

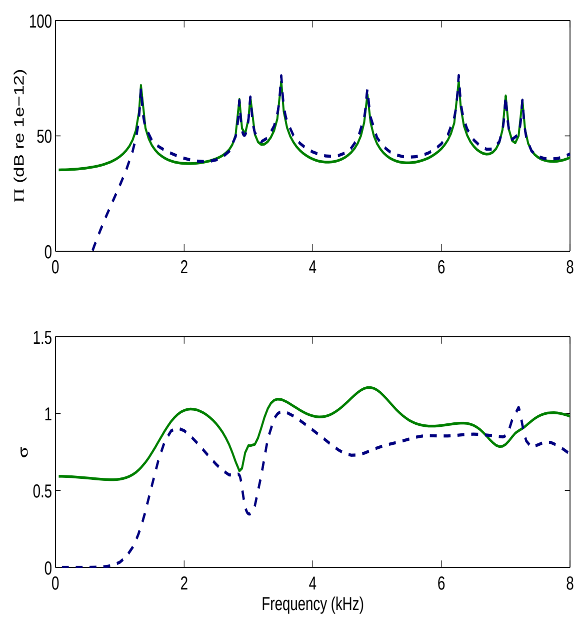


Figure 6: Π(ω) and σ(ω) for a combined excitation. Key: —, analytical calculation; ----, computed (FEM + BEM).

5. Conclusion

This article has introduced a new semi-analytical procedure for the calculation of sound radiation from modal and multi-modal vibrations of a thick annular disk. Based on a comparison with numerical and experimental results, it is evident that the proposed procedure has sufficient accuracy. Our procedure provides an efficient method of calculating modal and multi-modal sound radiations. Both out-of-plane and in-plane components of the disk vibration are included in the total sound radiation. Using this procedure, squeal noise radiation from brake rotor and the effect of modal interactions among adjacent components can be effectively analyzed. This subject is the focus of current work along with a determination of sound radiation from coupled modes.

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References

1. S. W. Kung, K. B. Dunlap, and R. S. Ballinger 2000 *SAE Technical Paper 2000-01-0444* Complex eigenvalue analysis for reducing low frequency brake squeal.
2. C. H Chung, W. Steed, K. Kobayashi, and H. Nakada 2000 *SAE Technical Paper 2000-01-1600* A new analysis method for brake squeal Part I: Theory for modal domain formulation and stability.
3. *I-DEAS User's manual version 8.2.* 2000 SDRC, USA.
4. A. W. Leissa 1987 *The Shock and Vibration Digest* **19**(3), 10-24. Recent Research and plate vibration, 1981-1985. Part 1: Classical theory.
5. O. G. McGee, C. S. Huang and A. W. Leissa 1995 *Int. J. Mech. Sci.* **37**(5), 537-566 Comprehensive exact solutions for free vibrations of thick annular plates with simply supported radial edges.
6. T. Irie, G. Yamada and Y. Muramoto 1984 *Journal of Sound and Vibration* **97**(1), 1711-175 Natural frequencies of in-plane vibration of annular plates.
7. H. Lee and R. Singh 2001 submitted to *Journal of Sound and Vibration* Acoustic radiation from radial modes of a thick annular disk, submitted to *Journal of Sound and Vibration*.
8. M. R. Lee and R. Singh 1994 *Journal of the Acoustical Society of America* **95**(6), 3311-3323. Analytical formulations for annular disk sound radiation using structural modes.
9. W. P. Rdzanek Jr. and Z. Engel 2000 *Applied Acoustics* **60**(5), 29-43. Asymptotic formula for the acoustic power output of a clamped annular plate.
10. H. LEE and R. SINGH 2002 submitted to *Journal of Sound and Vibration* Acoustic radiation from out-of-plane modes of an annular disk based on thick plate theory.
11. SYSNOISE User's manual Version 5.4. 1999 NIT, Belgium.