

# Effect of Engine Mount Damping on the Torque Roll Axis Decoupling

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## ABSTRACT

Several mounting system design concepts have been conceptually used to decouple the engine roll mode though limited success is observed in practice. One shortcoming of the existing theories is that they ignore damping in their formulations. To overcome this deficiency, we re-formulate the problem for a non-proportionally damped, linear system while recognizing that significant damping may be possible with passive (such as hydraulic), adaptive or active mounts. Only rigid body modes of power train are considered and chassis is assumed to be rigid. Complex mode method is employed and the torque roll axis (TRA) paradigms are re-examined in terms of mount rate ratios, mount locations and orientation angles. We will show that true TRA decoupling is not possible with non-proportional damping though it is theoretically achieved for a proportionally damped system. Results for both steady state (in the form of frequency response functions) and transient (given impulsive excitations) responses will be illustrated. The natural modes obtained using complex eigensolution method are coupled for the non-proportional damping case, even though they are completely decoupled for the proportional damping case. It is also seen that a higher value of non-proportionality induces more coupling between the rigid body motions of a powertrain. Our method and results are expected to lead to a better design of the mounting systems.

## INTRODUCTION

The torque roll axis (TRA) decoupling concept was investigated by Jeong and Singh [1]. Their analysis assumed a proportionally damped mounting system (from zero to moderate viscous damping ratios) when excited by the oscillating torque. Unlike other decoupling methods, their TRA decoupling method provided complete decoupling between roll and other motions. However, in reality the damping matrix is often non-proportional (assuming it is a linear system of

course). For instance, consider the case when one or two hydraulic mounts are combined with rubber mounts; the resulting damping matrix would be non-proportional since the effective viscous damping coefficient of a hydraulic mount is much higher than of a rubber mount [2]. In this paper, we consider the non-proportional damped mounting system and then extend the previous TRA study. This will allow us to comparatively evaluate undamped, proportionally damped, and non-proportionally damped systems. The degree of TRA decoupling will be quantified first in terms of the frequency response functions given unit harmonic torque. Then transient responses to an impulse torque input will be examined.

Figure 1 illustrates a typical powertrain isolation system that is composed of an inertial body, 3 or 4 mounts and a rigid foundation. The powertrain is assumed to be a rigid mass element of dimension 6 with time-invariant inertial properties. The resilient mounts are described by three tri-axial stiffness elements and they are assumed to be linear (insensitive to excitation amplitude). Each stiffness element is associated with viscous (or structural) damping characteristics. Further, it is assumed that the orientation of any mount can be arbitrarily adjustable to the desired direction. External or internal excitation forces are applied to the rigid inertial body. By assuming a rigid chassis, a 6-DOF linear time-invariant model is obtained. As a result of the assumptions made above, our model is limited to the lower frequency range. Over middle and higher frequency regimes, the powertrain body and chassis are expected to be compliant and the mounts could even exhibit the standing wave effect [3].

## ANALYTICAL FORMULATION

The following three coordinate systems are used in our work: Inertial coordinates  $(XYZ)_g$ , TRA coordinates  $(XYZ)_{TRA}$ , and mount (local) coordinates  $(XYZ)_{mi}$ .

The  $(XYZ)_g$  coordinate system is a ground-fixed reference frame with its origin at the static equilibrium (at the center of gravity, CG). The displacements of supported inertial body are assumed to be small and the motion vector  $q(X, Y, Z, \theta_x, \theta_y, \theta_z)$  can be completely expressed by the translational and angular displacements of the CG. The governing equations of motion are formulated in matrix form, as shown below, where  $q$ ,  $\dot{q}$ , and  $\ddot{q}$  are the displacement, velocity, and acceleration vectors, respectively of dimension 6:

$$\underline{M}\ddot{q} + \underline{C}\dot{q} + \underline{K}q = \underline{f} \quad (1)$$

Here  $\underline{M}$  is inertial (mass) matrix;  $\underline{K}$  is the stiffness matrix;  $\underline{C}$  is the viscous damping matrix (assuming non-proportional damping); and  $\underline{f}$  is the external excitation (force/torque) vector.

The localized stiffness,  $\underline{K}_{mi}$ , and damping matrices,  $\underline{C}_{mi}$ , in the local  $(XYZ)_{mi}$  coordinate systems at each mount are expressed as follows.

$$\underline{K}_{mi} = \begin{bmatrix} k_{pi} & 0 & 0 \\ & k_{qi} & 0 \\ sym. & & k_{ri} \end{bmatrix}, \quad \underline{C}_{mi} = \begin{bmatrix} c_{pi} & 0 & 0 \\ & c_{qi} & 0 \\ sym. & & c_{ri} \end{bmatrix} \quad (2a, 2b)$$

Here  $k_p$  is the principal compressive stiffness, and  $k_q$  and  $k_r$  are the principal shear stiffness components. Likewise, define  $c_p$  as the principal compressive damping, and  $c_q$  and  $c_r$  as the principal shear damping elements. Both matrices are transformed and expressed with respect to the global  $(XYZ)_g$  coordinate system

$(XYZ)_g$ : Inertial Coordinate System  
 $(XYZ)_{TRA}$ : TRA Coordinate System

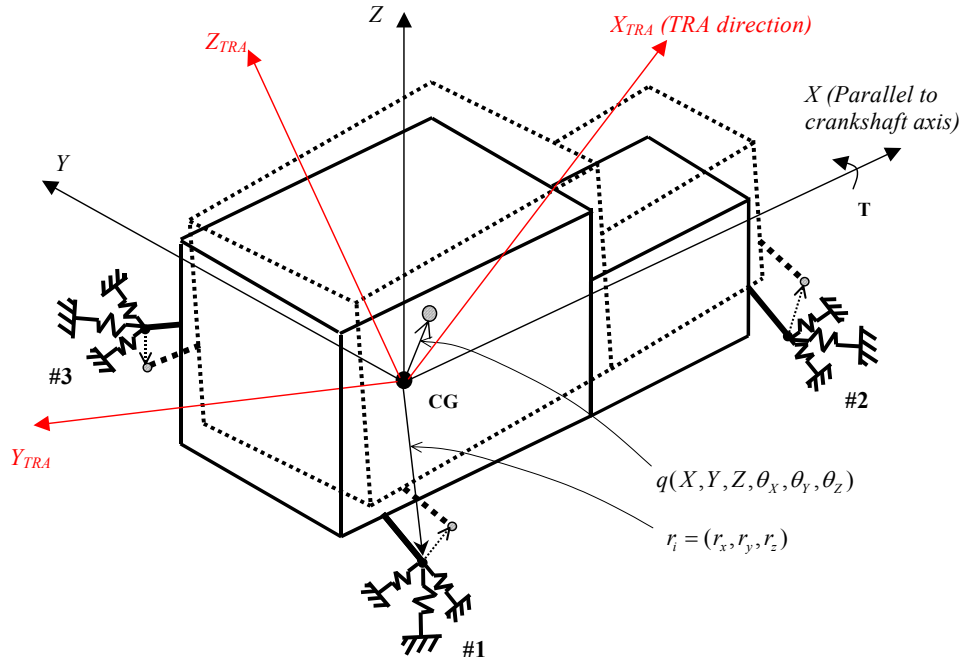


Fig. 1 Typical powertrain isolation system with inclined tri-axial stiffness elements. Both  $(XYZ)_g$  and  $(XYZ)_{TRA}$  are depicted but mount damping elements are not shown.

using a rotational matrix,  $\underline{\Theta}_{g,mi}$ , derived from the orientation angles.

$$\underline{K}_{g,mi} = \underline{\Theta}_{g,mi} \underline{K}_{mi} \underline{\Theta}_{g,mi}^T = \begin{bmatrix} k_{xxi} & k_{xyi} & k_{xzi} \\ & k_{yyi} & k_{yzi} \\ sym. & & k_{zzi} \end{bmatrix} \quad (3a)$$

$$\underline{C}_{g,mi} = \underline{\Theta}_{g,mi} \underline{C}_{mi} \underline{\Theta}_{g,mi}^T = \begin{bmatrix} c_{xxi} & c_{xyi} & c_{xzi} \\ & c_{yyi} & c_{yzi} \\ sym. & & c_{zzi} \end{bmatrix} \quad (3b)$$

Using the Euler angles as given by  $(\theta, \varphi, \phi)$  for i-th mount, the rotational matrix,  $\underline{\Theta}_{g,mi}$  is found. It includes a product of three rotational matrix operators as shown below.

$$\underline{\Theta}_{g,mi} = \begin{bmatrix} \cos\theta\cos\phi + \sin\phi\sin\theta\sin\varphi & \cos\phi\sin\phi & \cos\theta\sin\phi\sin\phi - \cos\phi\sin\theta \\ \cos\phi\sin\theta\sin\varphi - \cos\theta\sin\phi & \cos\theta\cos\phi & \cos\theta\cos\phi\sin\varphi + \sin\theta\sin\phi \\ \cos\phi\sin\theta & -\sin\varphi & \cos\theta\cos\phi \end{bmatrix} \quad (4)$$

Since the displacements at the mount location(s) caused by the rigid body rotations are computed by using a cross vector product, the resulting deflection,  $\underline{q}_{mi,t}$ , at each mount is as follows based on the rigid foundation assumption. Here the cross vector can be expressed by a tensor skew matrix,  $\underline{P}_{mi}$ .

$$\underline{q}_{mi,t} = \underline{q}_t + \underline{q}_\theta \times \underline{r}_{mi} \quad (5)$$

$$\underline{q}_{mi,t} = [\underline{I} \quad \underline{P}_{mi}] \underline{q} \quad (6)$$

$$\underline{P}_{mi} = \begin{bmatrix} 0 & r_{zi} & -r_{yi} \\ & 0 & r_{xi} \\ skew \ sym. & & 0 \end{bmatrix} \quad (7)$$

Here  $\underline{r}_{mi}(r_{xi}, r_{yi}, r_{zi})$  is the position vector of each mount and  $\underline{q}_t$  and  $\underline{q}_\theta$  are the translational and rotational displacements of powertrain (about CG), respectively. Translational and rotational reaction force  $\underline{f}_{mi,t}$  and moment  $\underline{f}_{mi,\theta}$  due to the vibratory displacement and velocity at each mount are calculated as follows.

$$\begin{aligned} \underline{f}_{mi,t} &= -\underline{K}_{g,mi} \underline{q}_{mi,t} - \underline{C}_{g,mi} \dot{\underline{q}}_{mi,t} \\ &= -[\underline{K}_{g,mi} \quad \underline{K}_{g,mi} \underline{P}_{mi}] \underline{q} - [\underline{C}_{g,mi} \quad \underline{C}_{g,mi} \underline{P}_{mi}] \dot{\underline{q}} \end{aligned} \quad (8)$$

$$\begin{aligned} \underline{f}_{mi,\theta} &= \underline{r}_{mi} \times \underline{f}_{mi,t} = \underline{P}_{mi}^T \underline{f}_{mi,t} \\ &= -[\underline{P}_{mi}^T \underline{K}_{g,mi} \quad \underline{P}_{mi}^T \underline{K}_{g,mi} \underline{P}_{mi}] \underline{q} - [\underline{P}_{mi}^T \underline{C}_{g,mi} \quad \underline{P}_{mi}^T \underline{C}_{g,mi} \underline{P}_{mi}] \dot{\underline{q}} \end{aligned} \quad (9)$$

Combining (8) and (9), we construct the global stiffness  $\underline{K}$  and damping  $\underline{C}$  matrices with respect to the inertial coordinate system for the powertrain mounting system with n number of mounts.

$$\underline{K} = \sum_{i=1}^n \underline{K}_i = \sum_{i=1}^n \begin{bmatrix} \underline{K}_{g,mi} & \underline{K}_{g,mi} \underline{P}_{mi} \\ \underline{P}_{mi}^T \underline{K}_{g,mi} & \underline{P}_{mi}^T \underline{K}_{g,mi} \underline{P}_{mi} \end{bmatrix} \quad (10a)$$

$$\underline{C} = \sum_{i=1}^n \underline{C}_i = \sum_{i=1}^n \begin{bmatrix} \underline{C}_{g,mi} & \underline{C}_{g,mi} \underline{P}_{mi} \\ \underline{P}_{mi}^T \underline{C}_{g,mi} & \underline{P}_{mi}^T \underline{C}_{g,mi} \underline{P}_{mi} \end{bmatrix} \quad (10b)$$

Eventually, we can assemble the governing equations in matrix form (1) with simplifying assumptions made.

### TRA DECOUPLING CONCEPT

To examine the roll mode decoupling, we transform the derived equations (in the inertial coordinates) to yet another set with respect to the TRA coordinate system. The torque roll direction is, however, required for this transformation. A torque roll axis is uniquely defined by both inertial properties and applied torque direction for an unconstrained rigid body with small motions given only one-dimensional dynamic torque [1]. When the mass matrix  $\underline{M}$  is given by (11a), the TRA direction  $\underline{q}_{TRA}$  will be expressed by (12a) with respect to the inertial coordinate system, as computed using the Euler's equations.

$$\underline{M} = \begin{bmatrix} \underline{M}_t & \underline{0} \\ \underline{0} & \underline{M}_\theta \end{bmatrix} \quad (11a)$$

$$\text{where, } \underline{M}_t = \begin{bmatrix} m & 0 & 0 \\ & m & 0 \\ sym. & & m \end{bmatrix}, \underline{M}_\theta = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ & I_{yy} & -I_{yz} \\ sym. & & I_{zz} \end{bmatrix} \quad (11b, c)$$

and,

$$\underline{q}_{TRA} = \begin{bmatrix} \underline{q}_{TRA,t} \\ \underline{q}_{TRA,\theta} \end{bmatrix} \quad (12a)$$

where,

$$\underline{q}_{TRA,t} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \underline{q}_{TRA,\theta} = \underline{M}_\theta^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} I_{11} & & sym. \\ I_{21} & I_{22} & \\ I_{31} & I_{32} & I_{33} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} I_{11} \\ I_{21} \\ I_{31} \end{bmatrix} \quad (12b, c)$$

We construct a new TRA coordinate system where one axis is parallel to  $\underline{q}_{TRA}$  using the transformation matrix

$\underline{\Theta}_{TRA,g}$  as derived from the orthogonality property of corresponding axes. As a result, the governing equations in the TRA coordinate system are constructed as shown below.

$$\underline{M}'' \ddot{\underline{q}}'' + \underline{C}'' \dot{\underline{q}}'' + \underline{K}'' \underline{q}'' = \underline{f}'' \quad (13a)$$

where,

$$\underline{M}'' = \begin{bmatrix} \underline{\Theta}_{TRA,g} \underline{M}_t \underline{\Theta}_{TRA,g}^T & \underline{0} \\ \underline{0} & \underline{\Theta}_{TRA,g} \underline{M}_\theta \underline{\Theta}_{TRA,g}^T \end{bmatrix} \quad (13b)$$

$$\underline{K}'' = \sum_{i=1}^n \begin{bmatrix} \underline{\Theta}_{TRA,g} \underline{K}_{g,mi} \underline{\Theta}_{TRA,g}^T & \underline{\Theta}_{TRA,g} \underline{K}_{g,mi} P_{mi} \underline{\Theta}_{TRA,g}^T \\ \underline{\Theta}_{TRA,g} P_{mi}^T \underline{K}_{g,mi} \underline{\Theta}_{TRA,g}^T & \underline{\Theta}_{TRA,g} P_{mi}^T \underline{K}_{g,mi} P_{mi} \underline{\Theta}_{TRA,g}^T \end{bmatrix} \quad (13c)$$

$$\underline{C}'' = \sum_{i=1}^n \begin{bmatrix} \underline{\Theta}_{TRA,g} \underline{C}_{g,mi} \underline{\Theta}_{TRA,g}^T & \underline{\Theta}_{TRA,g} \underline{C}_{g,mi} P_{mi} \underline{\Theta}_{TRA,g}^T \\ \underline{\Theta}_{TRA,g} P_{mi}^T \underline{C}_{g,mi} \underline{\Theta}_{TRA,g}^T & \underline{\Theta}_{TRA,g} P_{mi}^T \underline{C}_{g,mi} P_{mi} \underline{\Theta}_{TRA,g}^T \end{bmatrix} \quad (13d)$$

$$\underline{f}'' = \underline{\Theta}_{TRA,g} \underline{f}_t + \underline{\Theta}_{TRA,g} \underline{f}_\theta \quad (13e)$$

The “true” TRA mode decoupling is achieved by letting the TRA direction be one of the natural modes [1]. Essentially, the corresponding eigenvalue problem for an undamped mounting system must be satisfied.

$$\underline{K}'' \underline{q}_{TRA} = \lambda \underline{M}'' \underline{q}_{TRA} \quad (14)$$

Mounting system parameters such as the orientation angles, stiffness ratios ( $k_p / k_r$ ), and their locations (as given by the position vectors) could be adjusted for this purpose.

The resulting modes from the eigenvalue problem (14) for a proportionally damped system will yield the same eigenvectors as those of an undamped system. Using this property, we can prove that the TRA decoupling for a proportionally damped system is achieved as follows. In the governing equations (1), apply the harmonic torque excitation as  $\underline{f} = \underline{T}_a e^{i\omega t}$  and then the steady-state response will be in the form of  $\underline{q} = \underline{q}_a e^{i\omega t}$ .

$$-\omega^2 \underline{M} \underline{q}_a + i\omega \underline{C} \underline{q}_a + \underline{K} \underline{q}_a = \underline{T}_a \quad (15)$$

Based on the orthogonal property of eigenvectors and the fact that both undamped and proportionally damped systems have same eigenvectors, the dynamic response of the 6-DOF system is expressed by the following equation.

$$\underline{q}_a = b_1 \underline{u}_1 + b_2 \underline{u}_2 + b_3 \underline{u}_3 + b_4 \underline{u}_4 + b_5 \underline{u}_5 + b_6 \underline{u}_6 \quad (16)$$

where,  $\underline{u}_i$  are eigenvectors and  $b_i$  are the modal participation coefficients. Orthogonality property of the eigenvectors provides the following relations.

$$\underline{u}_i^T \underline{M} \underline{u}_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases} \quad (17a)$$

$$\underline{u}_i^T \underline{K} \underline{u}_j = \begin{cases} 0 & \text{if } i \neq j \\ k_i & \text{if } i = j \end{cases} \quad (17b)$$

Multiply equation (15) by  $\underline{u}_i^T$  and use equations (16) and (17a, b) to achieve,

$$-\omega^2 b_i + i\omega \underline{u}_i^T \underline{C} \sum_{j=1}^6 b_j \underline{u}_j + b_i k_i = \underline{u}_i^T \underline{T}_a \quad (18)$$

For the proportionally damped, assume that  $\underline{C} = \alpha \underline{M} + \beta \underline{K}$  where  $\alpha$  and  $\beta$  are arbitrary scalar values. This allows us to write (18) as:

$$-\omega^2 b_i + i\omega(\alpha + \beta k_i) b_i + k_i b_i = \underline{u}_i^T \underline{T}_a \quad (19)$$

$$b_i = \frac{\underline{u}_i^T \underline{T}_a}{-\omega^2 + i\omega(\alpha + \beta k_i) + k_i} \quad (20)$$

On the other hand, one of the modes should be parallel to the TRA direction for the roll mode motion decoupling. It is achieved by adjusting the mounting parameters. Define the TRA direction,  $\underline{q}_{TRA}$ , and let one mode,  $\underline{u}_j$ , be in the TRA direction as follows. Here  $a$  and  $c$  are constants.

$$\underline{q}_{TRA} = a \underline{M}^{-1} \underline{T}_a \quad (21)$$

$$\underline{u}_j = c \underline{q}_{TRA} \quad (22)$$

Combining equations (21) and (22), we obtain

$$\underline{u}_i^T \underline{T}_a = \frac{\underline{u}_i^T \underline{M} \underline{u}_j}{ca} \begin{cases} = 0 & \text{if } i \neq j \\ \neq 0 & \text{if } i = j \end{cases} \quad (23)$$

From equations (20) and (23), only  $b_j \neq 0$  and  $b_i = 0$ . Eventually, the motion response,  $\underline{q}$ , exists only in the TRA direction for a proportionally damped dynamic system.

Non-proportionally damped system exhibits complex modes that could differ from those for proportionally damped system (including the one with zero damping). This implies that the modes of a non-proportionally

damped system have arbitrary phase angles; conversely, the modes of a proportionally damped system have only in-phase or out-of-phase values. From this reason, it is expected that the TRA decoupling method (as derived above) would no longer be valid for the non-proportional damping case. One example will be given in the following section to show that TRA decoupling method cannot be applied to non-proportionally damped system for the purpose of roll mode decoupling.

## RESULTS AND DISCUSSION

Our example case is a V6 diesel engine [4] whose mounting system parameters (stiffness values, mount locations and orientation angles) are given in Tables 1, 2, and 3. The motion analysis is conducted by using the TRA decoupling method, first for proportional and then non-proportional damping. Only the dynamic torque excitation is considered and unit harmonic torque is assumed. Frequency response functions are calculated, as shown in Figs. 2 and 3, for two cases: TRA decoupling method with proportional damping and TRA method with non-proportional damping. Spectra for the original mounting system (without the TRA decoupling design) are also presented for the sake of comparison.

Table 1 Mount stiffness values for the example case

Mount #	Stiffness (N/mm)		
	$k_p$	$k_q$	$k_r$
1	224	45	45
2	170	126	49
3	217	434	109
4	232	464	116

Table 2 Mount locations for the example case

Mount #	Location (mm)		
	$r_x$	$r_y$	$r_z$
CG	0	0	0
1	-225	-309	-199
2	361	-282	-251
3	-195	141	229
4	293	167	-245

Table 3 Mount orientation angles for the example case

Mount #	Orientation (degree)		
	$\theta$	$\varphi$	$\phi$
1	0	-45	0
2	0	-39	180
3	0	-75	0
4	0	-45	180

Next, an impulse torque is applied and the resulted transient responses are shown in Figs. 4 and 5. It is clearly seen that the complete motion decoupling of a mounting system (that is originally coupled to begin with) is achieved by selecting appropriate parameters that yield the decoupled motions with proportional damping. Observe that for a proportionally damped system, motion exists only in the roll direction  $\theta_x$  as shown in Fig. 4. Observe that the decoupled motions are coupled again with an introduction of non-proportional damping as shown in Fig. 5 (and when compared with the spectra of Fig. 4).

Modal analysis is also conducted by using the complex eigensolution method [5]. Table 4 shows the results of two modes using real and complex eigensolutions. Modal coupling in the roll and vertical bounce modes is seen for the non-proportional damping; conversely, it does not exist in the proportional damping case. Two resonant peaks in the vertical bounce (z direction) spectra are also found in Fig. 3; both peaks are due to this modal coupling.

Table 4 Modal analysis for the example case with two damping cases

Dominant mode	Roll $\theta_x$		Vertical X (Bounce)		
	Prop.	Non-prop.	Prop.	Non-prop.	
Modal displacements* (magnitudes only)	X	0	0.02	0	0.01
	Y	0	0.10	0	0.54
	Z	0	0.02	1	1.00
	$\theta_x$	1	1.00	0	0.52
	$\theta_y$	0	0.27	0	0.05
	$\theta_z$	0	0.01	0	0.01
Viscous damping ratio (%)	2.1	3.1	2.5	4.6	

Key \*: Units of relative X, Y, and Z are mm; units of  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  degrees.

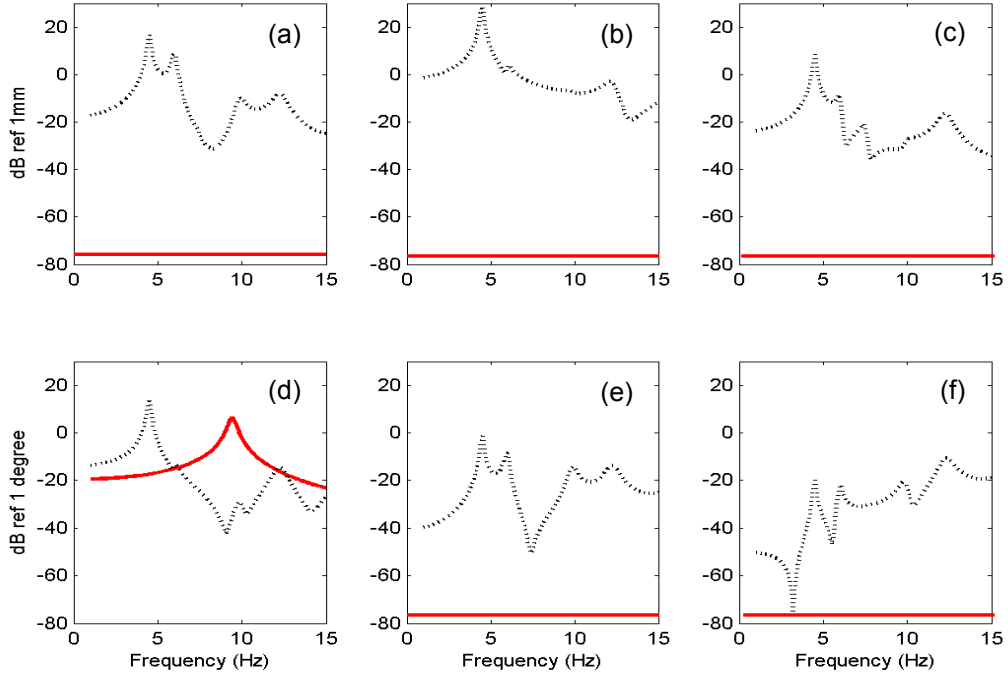


Fig. 2 Frequency response functions with proportional damping given unit harmonic torque. (a) X; (b) Y; (c) Z; (d)  $\theta_X$ ; (e)  $\theta_Y$ ; (f)  $\theta_Z$ . Key: ---, original mounting system; —, TRA decoupled mounting system with proportional damping.

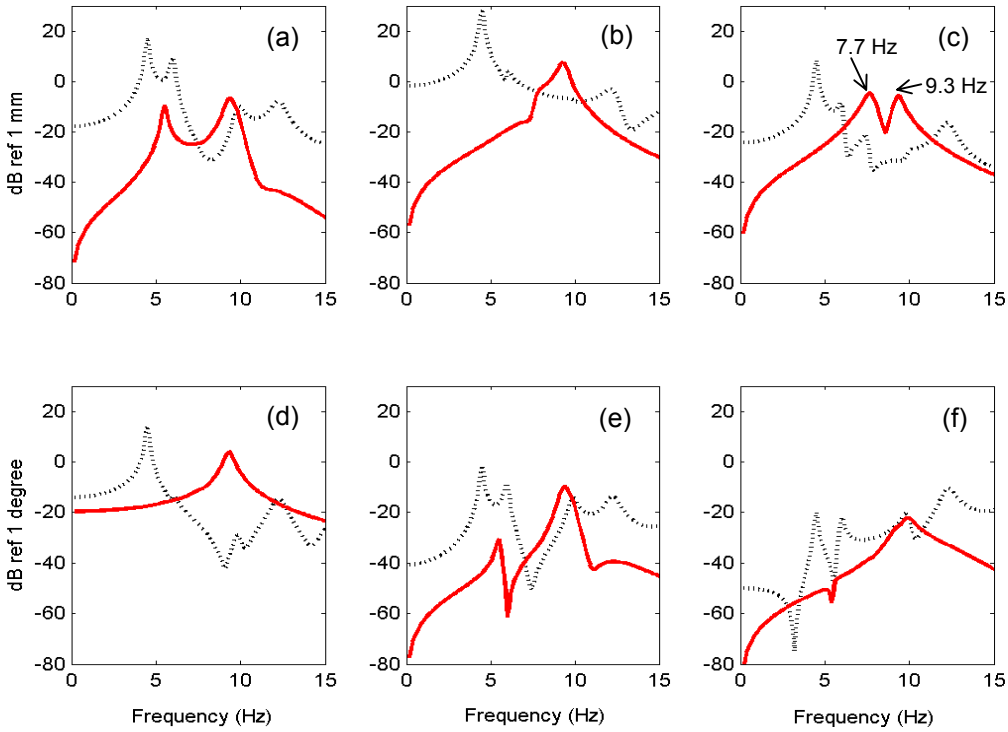


Fig. 3 Frequency response functions with non-proportional damping given harmonic torque. (a) X; (b) Y; (c) Z; (d)  $\theta_X$ ; (e)  $\theta_Y$ ; (f)  $\theta_Z$ . Key: ---, original mounting system; —, TRA mounting system with non-proportional damping.

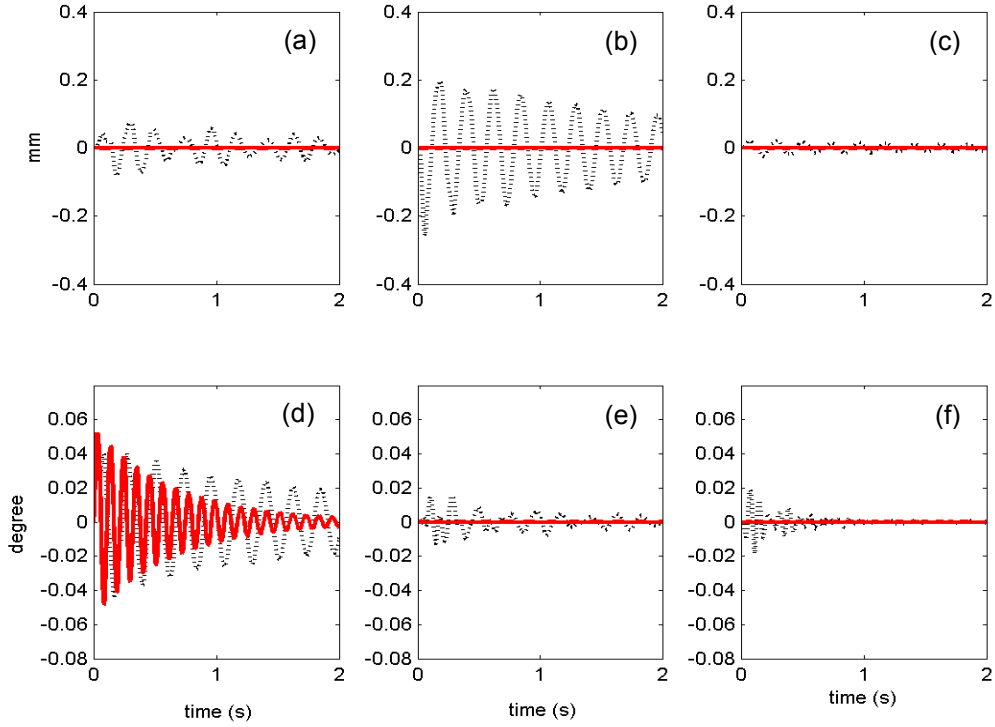


Fig. 4 Impulsive responses with proportional damping given torque impulse. (a) X; (b) Y; (c) Z; (d)  $\theta_X$ ; (e)  $\theta_Y$ ; (f)  $\theta_Z$ . Key: - - -, original; —, TRA with proportional damping.

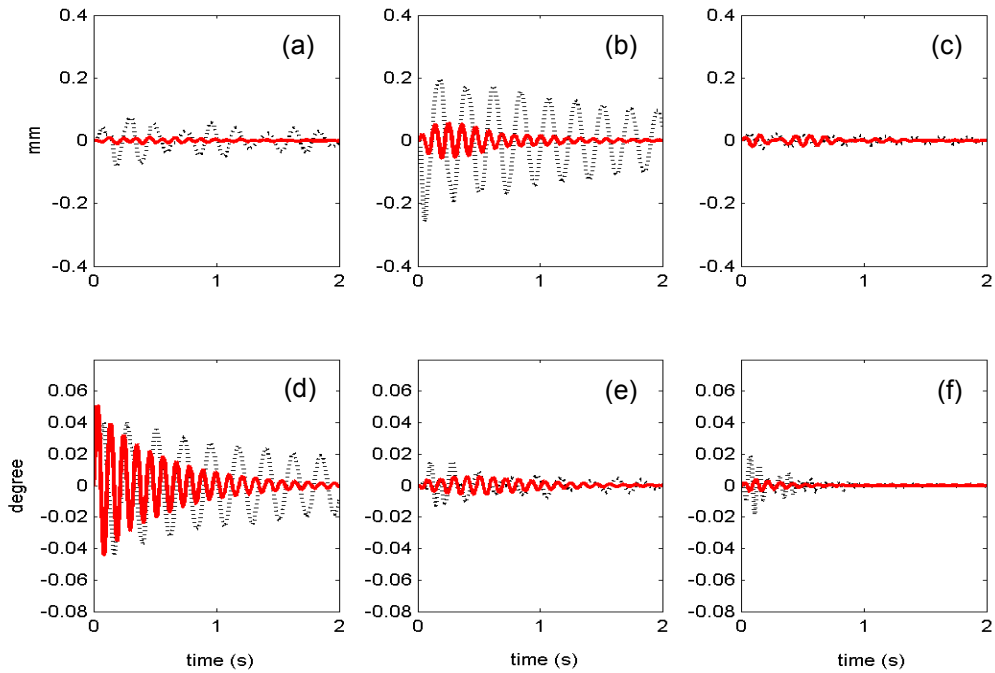


Fig. 5 Impulsive responses with non-proportional damping given torque impulse. (a) X; (b) Y; (c) Z; (d)  $\theta_X$ ; (e)  $\theta_Y$ ; (f)  $\theta_Z$ . Key: - - -, original; —, TRA with non-proportional damping.



Caughey and O'Kelly [6] derived the following condition for the general case of proportional damping:  $\underline{CM}^{-1}\underline{K} = \underline{KM}^{-1}\underline{C}$ . Using this, Nair and Singh [7] developed an index,  $\sigma$ , for quantifying the non-proportionality in discrete vibratory systems:

$$\underline{E} = \underline{CM}^{-1}\underline{K} - \underline{KM}^{-1}\underline{C} \quad (24)$$

$$\sigma = \sum_{i=1}^n \sum_{j=1}^n |E_{ij}|. \quad (25)$$

This non-proportionality index,  $\sigma$ , is used in our work for quantifying the extent of non-proportional damping. It is intentionally varied from a proportionally damped to highly non-proportionally damped system. Comparative spectra are shown in Fig. 6 where the damping in one of the mounts is changed by 1, 3, and 10 times the nominal (proportional) damping value respectively. The non-proportionality index is normalized with respect to the maximum value. It is seen that a higher value of  $\sigma$  suggests more coupling between the rigid body motions of a powertrain.

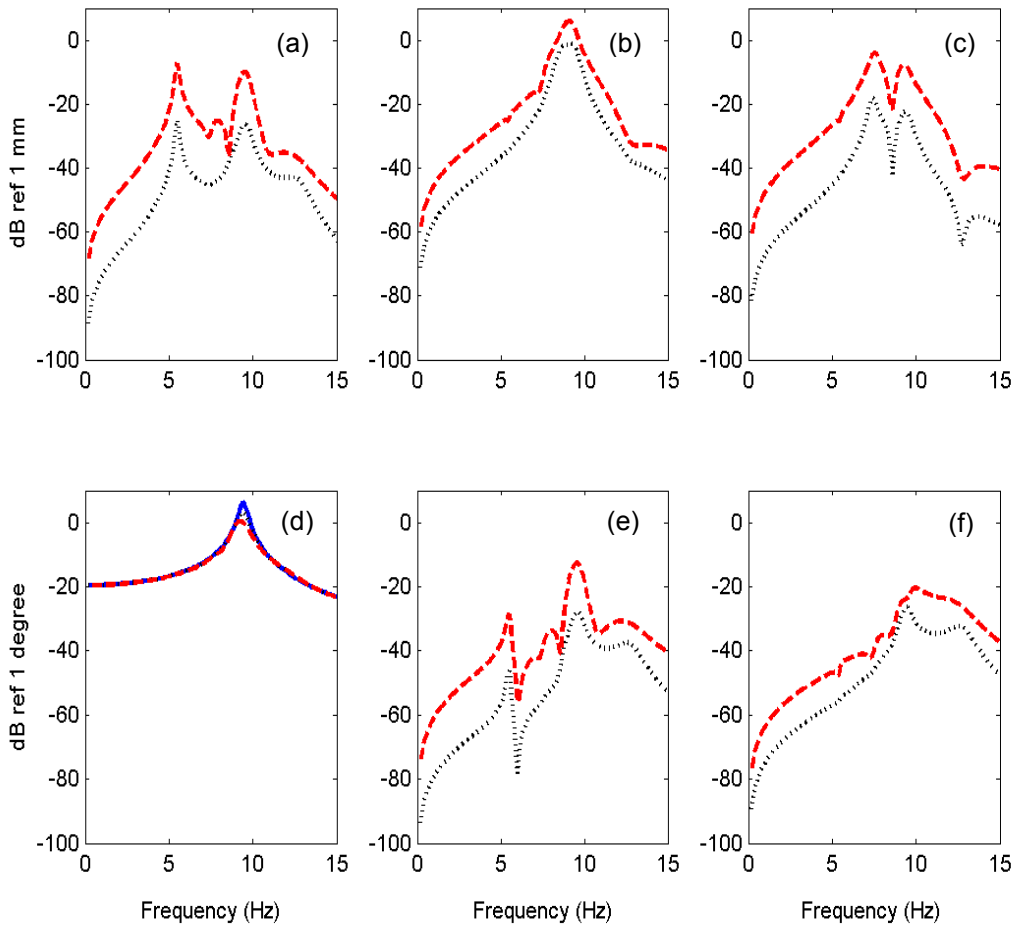


Fig. 6 Frequency response functions with three values of  $\sigma$  given harmonic torque excitation. (a) X; (b) Y; (c) Z; (d)  $\theta_X$ ; (e)  $\theta_Y$ ; (f)  $\theta_Z$ . Key: —,  $\sigma = 0$  (proportional); ---,  $\sigma = 0.24$  (non-proportional); - · -,  $\sigma = 1$  (non-proportional).



## CONCLUSION

For a proportionally damped mounting system, an analytical proof is provided that shows the torque roll axis mode decoupling is possible when excited by only the oscillating torque. But, with an introduction of non-proportional damping (in the mounting system), the TRA decoupling is no longer achieved. Frequency response functions and impulsive responses for the example powertrain mounting system clearly show the coupled motions. It is also seen that the natural modes are coupled for the non-proportional damping case, even though they are completely decoupled for the proportional damping case. This knowledge illustrates difficulties encountered in designing real-life systems. Further complications arise from spectrally-varying and amplitude sensitive stiffness and damping values. This topic is being investigated.

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