Dynamic analysis of flexible gears for high power density transmissions

H.Vinayak, R.Singh
The Ohio State University, USA

SYNOPSIS Free and forced vibration characteristics of a spur gear pair system are examined for both rigid and flexible gear blank cases. The gear blank itself is also analyzed by using Rayleigh-Ritz and finite element methods, and predictions match measured modal data. Dynamic interactions between gear blanks and gear mesh-shaft system are studied. Directions for further research are identified.

1. INTRODUCTION

Thin, weight optimized gear blanks are common in many applications. The natural frequencies of such compliant gears may lie within the gear mesh frequency excitation regimes. However, gear dynamics researchers have mostly focused on mathematical models containing rigid gears [1,2] which is also evident from the literature reviews conducted by Ozguven and Houser [3] and later by Blakenship and Singh [4]. Among the limited studies dealing with flexible gear blanks includes an effort by Amirouche *et al.* [6] who used a hybrid finite element method. There are at least two modeling issues which need to be addressed when dealing with flexible gears. First, how should one include gear blank flexibility coordinates in an overall model of the transmission system consisting of flexible shafts, bearings and other components? Second, what is the extent of dynamic coupling or interaction between the flexural modes of gear blanks and the bending-torsional dynamics associated with gear mesh and shaft systems? This paper examines both in a conceptual manner while considering a simple system consisting of a spur gear pair. Only modal characteristics and frequency response functions are examined over a limited frequency range (say upto 4 kHz) and hence all formulations are assumed to be linear time-invariant type [2,4]. This analysis is a prelude to a more comprehensive dynamic investigation of high power density transmissions which is underway.

The mathematical formulation introduced in this paper will be included in the MMM (Multi-Mesh gear analysis using Multi-body dynamics techniques) software that has been discussed in an earlier paper by Vinayak and Singh [5].

2. FORMULATION

Figure 1(a) shows the schematic of an example case consisting of two flexible spur gears of speed ratio μ, flexible shafts and compliant bearings. Each gear-shaft system is represented by six rigid degrees and a few flexible degrees of freedom; the number of latter depends on the desired accuracy. In this paper, a three dimensional distributed contact stiffness is used to describe the meshing interface between the gears [11]. Typically, this stiffness is time as well as gear rotational position dependent but an eigenvalue problem can be posed if a time-averaged stiffness is used. It can be obtained from the existing gear kinematics and contact analysis programs [7].The gear blank dynamics is included in the gear-shaft model by introducing additional flexible coordinates. Due to the space limitations, mathematical treatment of this procedure is omitted from this paper; it can be found in reference [11]. A few alternate methods can be used to obtain the gear blank flexibility coordinates. If a gear resembles a thin disk with classical boundary conditions [12], exact solutions of the classical annular plate theory [8,9] may be used. However, a gear is typically mounted on a flexible shaft which imposes a non-classical boundary condition at the inner edge. Further, the gear blank itself may be of irregular geometry and may contain holes and stiffeners or ribs. For such cases the closed form solutions consisting of Bessel's functions become cumbersome and computationally tedious. Consequently, approximate or numerical techniques such as Rayleigh-Ritz [10] or finite element methods must be employed. A finite element model of the example case of Figure 1 has been developed by using 3-D isoparametric elements for the flexible gears and shafts. Bearings are modeled as radial stiffness elements. The gear mesh stiffness is represented by a linear array of springs connecting the two gears and is based on the average mesh stiffness obtained from the contact analysis program [7].

A separate analytical and experimental study was conducted on a gear blank which is simulated as an equivalent thin annular disk as shown in Figure 2. This disk was suspended in the free-free mode and all modes upto 4 kHz were measured and predicted by using the finite element method (FEM) and the Rayleigh-Ritz procedure [9] where the admissible functions for the gear blank are

$$u_z(r,\theta) = \sum_{i,j} \Theta_{ij} R_i;$$

$$\Theta_{ij} = \left(a_{ij}\cos(j\theta) + b_{ij}\sin(j\theta)\right);$$

$$R_i = (r - r_s)^i (r_o - r_s)^{-i};$$

$$u_r(r,\theta,z) = -z\frac{\partial u_z}{\partial r};\quad u_\theta(r,\theta,z) = -z\frac{\partial u_z}{r\partial\theta};$$

$$i = 0,1,2,\ldots;\; j = 0,1,2,\ldots \qquad (1a-e)$$

where u_r, u_θ and u_z are deformations in the r, θ and z directions and r_s and r_o are inner and outer radii as shown in Figure 2. Here a_{ij} and b_{ij} are the modal coefficients for the gear blank.

3. RESULTS

3.1 **Gear Blank Modes** : Both Rayleigh-Ritz [10] and finite element methods are used to calculate natural frequencies and associated mode shapes of a gear blank with free inner and outer edges. Table 1 compares predictions with measured natural frequencies. Observe a close match between theory and experiment. Similarly the experimental and finite element mode shapes compare well in Figure 3 with the results obtained from the Rayleigh-Ritz method. Table 1 also shows the natural frequencies of the second gear blank of Figure 1(a). The first two frequencies for each blank are the repeated roots with two nodal diameters (m=2). Figure 3(a) shows this mode of vibration for the gear blank #1 at $r=r_S$. Figure 3(b) shows the deformation corresponding to the mode (0,1) along a radial line given $\theta = 0^o$. The next mode is again a repeated eigenvalue with three nodal diameters (m=3) as shown in Figure 3(c). Since the Rayleigh-Ritz results are very close to both measured and finite element mode shapes, these may form the basis for further analysis. It is clear from Table 2 which summarizes natural frequencies for an annular plate with clamped inner edge (r_s) while the outer edge (r_o) is free, that a non-classical boundary condition exists at $r = r_S$.

3.2 **Flexible Geared System**: Finite element analyses were performed for the system of Figure 1, once with rigid and then with flexible gears. Two systems with unity (μ=1.0) and nonunity (μ=1.2) gear ratios were used in the analysis. Table 3 lists the natural frequencies of all four cases. Figure 4(a) shows the driving-point dynamic compliance $C_{PP}(f)$ spectra of a unity ratio geared system, where C_{PP}(μm/N) = Displacement at P / Force at P. The sinusoidal force and response locations are at point P ($r = r_{O1}$ and $\theta = 140^o$ for gear #1) as shown in Figure 1(b). As evident from the C_{PP} spectra, resonant peaks corresponding to the gear blanks have significant modal participations. Also the coupling between the rigid and flexible modes seems to reduce the natural frequencies corresponding to the gear mesh and shaft dynamics. The same phenomenon is observed in Figure 4(b) which shows the cross-point dynamic compliance spectra $C_{QP}(f)$, given sinusoidal force at point P and response at point Q ($r = r_{O2}$ and $\theta = 300^o$ for gear #2) as shown in Figure 1(b). Now, the blank modes have almost same participation levels as the rigid body type modes. Both driving-point and cross-point spectra for a gear pair with μ = 1.2 are also shown in Figure 4. It is seen that the system now has a higher modal density which results mainly due to different resonant frequencies of both gear blanks. The cross-spectrum of Figure 4(b) for μ = 1.2 also shows splitting in some of the rigid body frequencies when compared to the case with μ = 1.0. This results from the physical asymmetry of the system.

4. CONCLUSION

Two observations can be drawn from this modal study of a simplified system. First, the gear blank dynamics is very important when modeling high speed transmissions containing thin flexible gears since various dynamic interactions are seen along with an increase in the modal density over the applicable frequency range of interest. Second, analytical admissible functions may be used to analyze the gear blanks by employing the Rayleigh-Ritz technique. Such shape functions may have to be modified for irregular geometry and non-classical boundary conditions. This work is in progress. Also, efforts are underway to develop an analytical methodology to express the gear blank flexibility in the multi-body dynamics format [11]. Finally, the software developed will be used to analyze multi-mesh geared systems.

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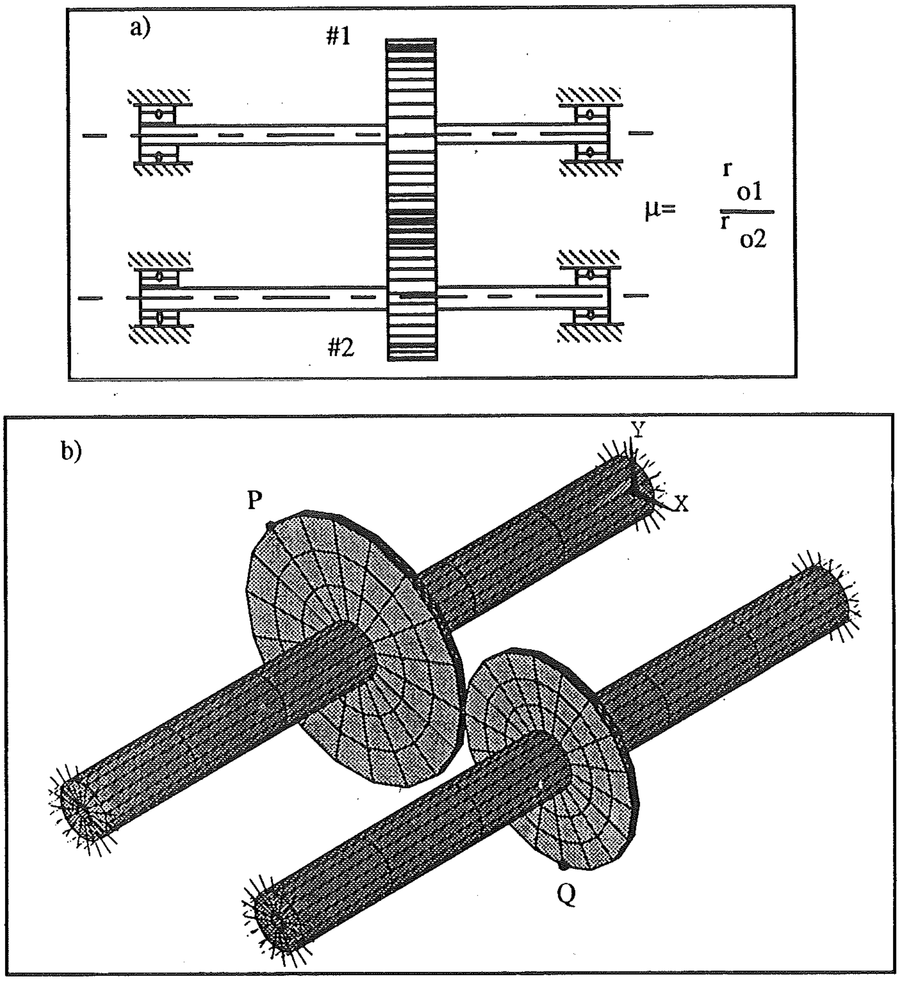


Figure 1. Spur gear pair example. a) Schematic and b) Finite element model.

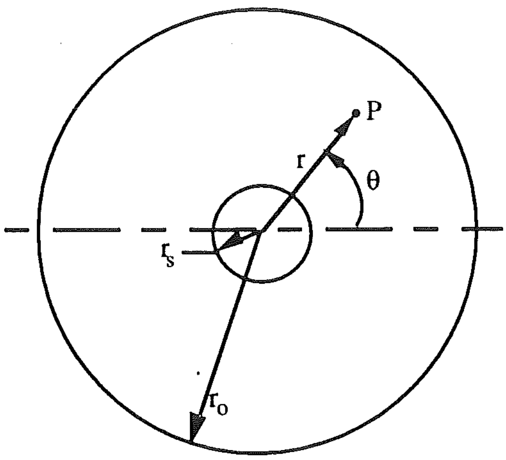


Fig. 2. Schematic of the gear blank of thickness t; here the gear blank has been simulated as an annular disk.

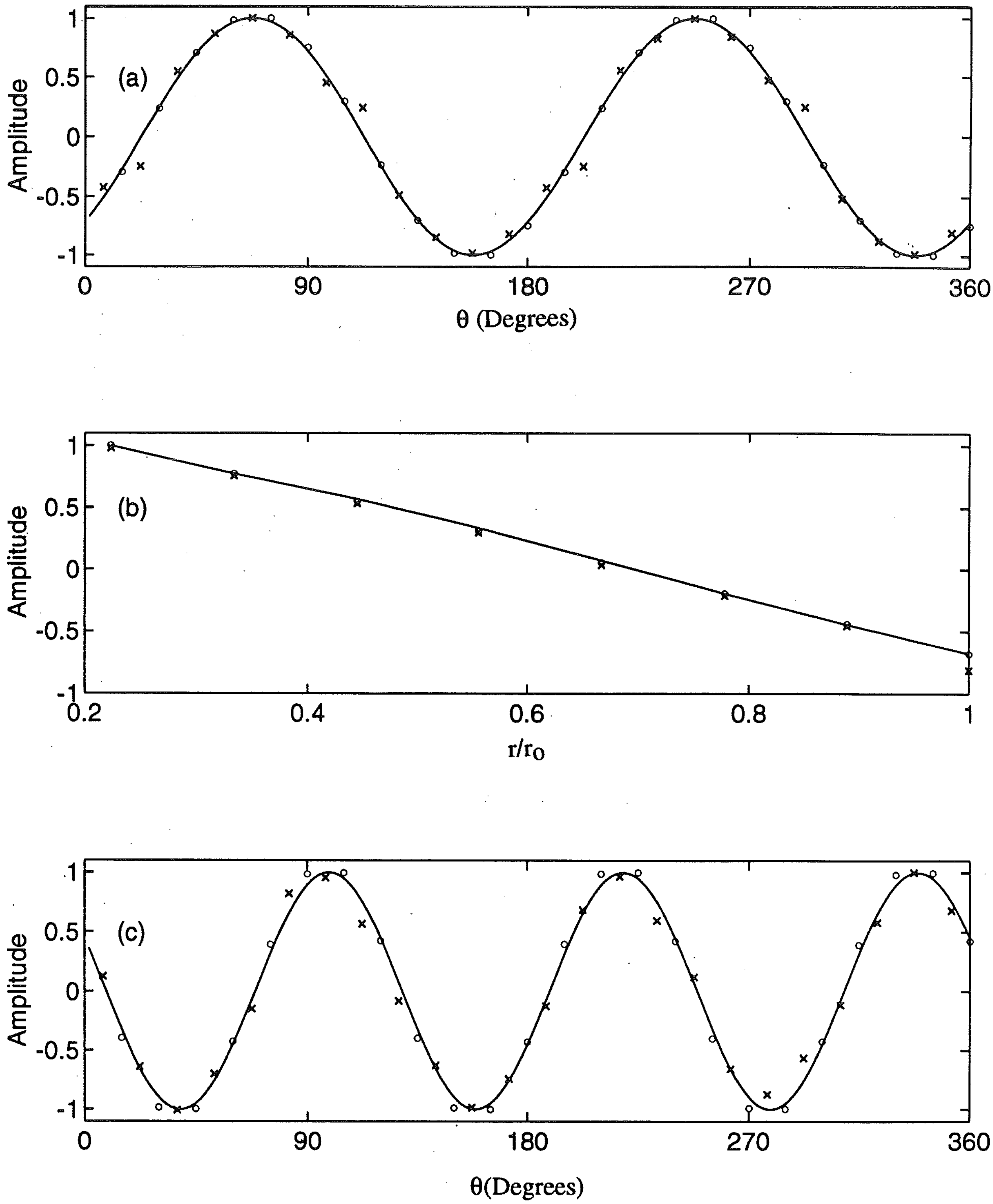
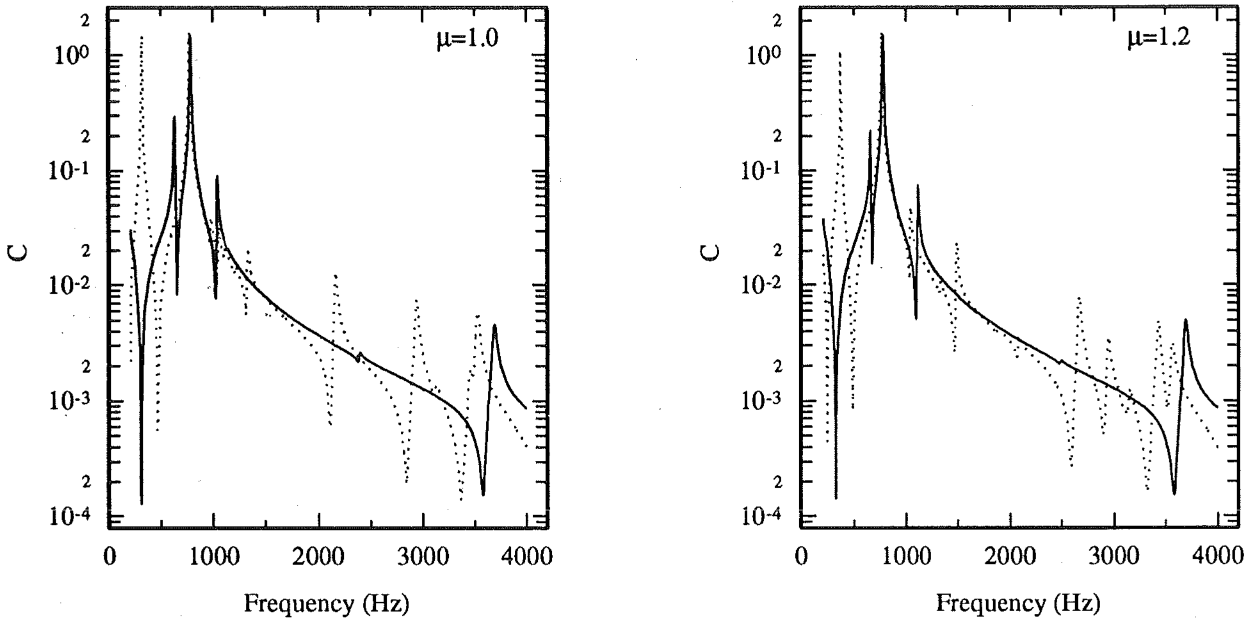
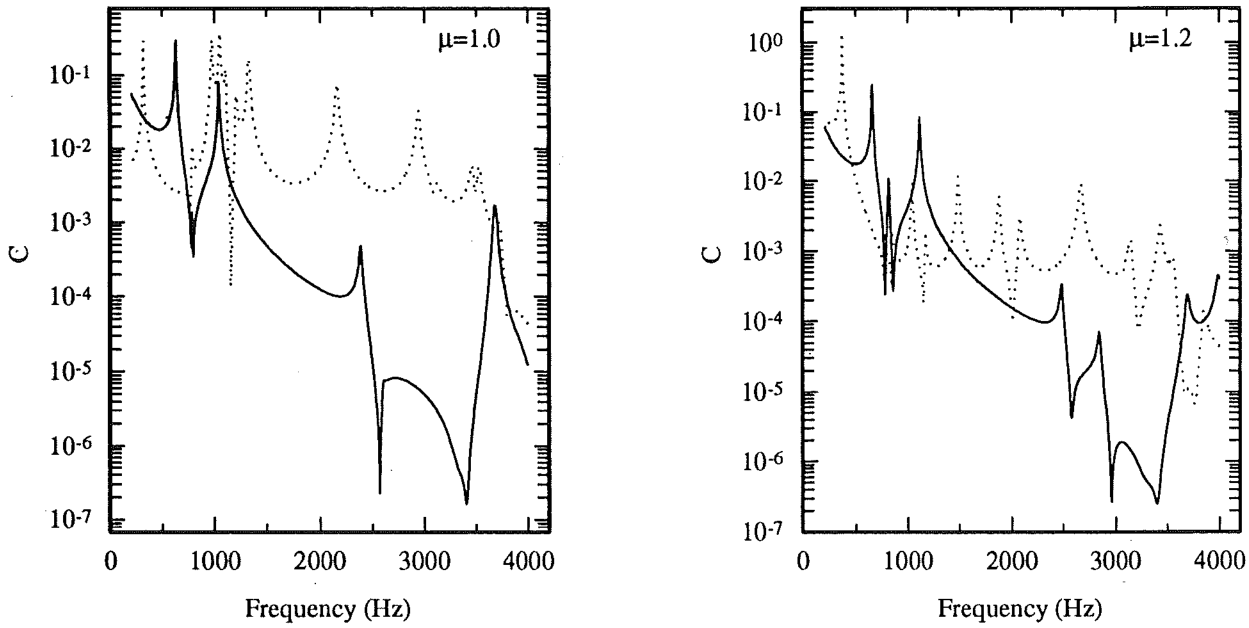


Figure 3. Comparison of gear blank (#1) mode shapes with free-free boundary conditions. Key: —— Rayleigh-Ritz, x Experimental, o Finite element method. (a) Mode (2,0) at $r = r_0$, (b) Mode (0,1) at $\theta = 0^0$ and (c) Mode (3,0) at $r = r_0$.



a) Driving point $C_{PP}(f)$



b) Cross point $C_{QP}(f)$

Figure 4. Dynamic compliance spectra predicted by the finite element method. Key: ——— System with rigid gears, ······· System with flexible gears. Units of C are μm/N.

Table 1. Natural frequencies of gear blank of Figure 2 with free-free boundary conditions

	Blank #1			Blank #2	
r_s	20 mm			20 mm	
r_o	90 mm			75 mm	
t	6.4 mm			6.4 mm	
Mode	Natural Frequencies (Hz)				
m,n	Expt.	Ritz	FEM	Ritz	FEM
2,0	984	978	978	1517	1534
2,0	984	978	972	1517	1534
0,1	1600	1595	1602	2531	2569
3,0	2370	2374	2371	3762	3811
3,0	2370	2374	2380	3762	3811
1,1	3620	3688	3627		
1,1	3620	3688	3650		

m = number of nodal diameters
n = number of nodal circles

Table 2. Natural frequencies of gear blank of Figure 2 with fixed-free boundary conditions

	Blank #1	
r_s	20 mm	
r_o	90 mm	
t	6.4 mm	
Mode	Natural Frequencies (Hz)	
m,n	Expt.	Ritz
1,0	765	777
1,0	774	777
0,0	908	903
2,0	1190	1132
2,0	1191	1132
0,1	2209	2034

m = number of nodal diameters
n = number of nodal circles

Table 3. Predicted natural Frequencies of gear-shaft system

Mode	Natural Frequencies (Hz)			
	Rigid Gears		Flexible Gears	
	$\mu=1.0$	$\mu=1.2$	$\mu=1.0$	$\mu=1.2$
1	0	0	0	0
2	620	658	375	372
3	780	780	766	766
4	780	817	766	774
5	780	853	767	837
6	1037	1114	777	845
7	2387	2389	969	970
8	2389	2481	969	1039
9	2389	2768	970	1172
10	2598	2843	1049	1326
11	3430	3430	1103	1484
12	3439	3439	1201	1764
13	3668	3685	1324	1874
14	3690	3993	1324	2080
15			1327	2261
16			2164	2662
17			2936	2937
18			2937	3089
19			2940	3117
20			3091	3152
21			3094	3166
22			3112	3418
23			3125	3426
24			3429	3460
25			3461	3550
26			3492	3845
27				3523
28				3734